UNIT–V : LINEAR PROGRAMMING

CHAPTER 12

Term-I

LINEAR PROGRAMMING

Syllabus

- *Introduction, related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems, graphical method of solution for problems in two variables, feasible and infeasible region (bounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).*

STAND ALONE MCQs (1 Mark each)

Q. 1. The corner points of the feasible region determined by the system of linear constraints are (0, 0), (0, 40), (20, 40), (60, 20), (60, 0). The objective function is

$$Z = 4x + 3y.$$

Compare the quantity in Column A and Column B

Column A	Column B
Maximum of Z	325

(A) The quantity in column A is greater.
(B) The quantity in column B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined on the basis of the information supplied.

Ans. Option (B) is correct.

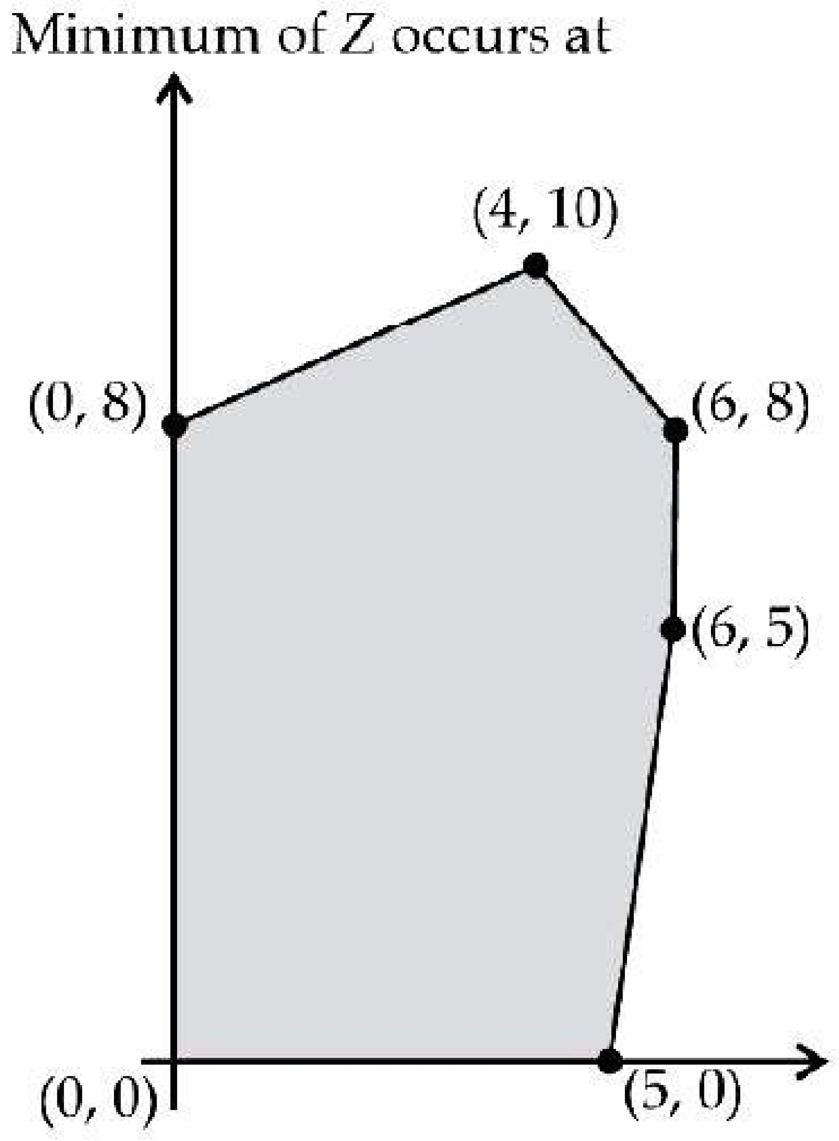
Explanation :

Corner points	Corresponding value of $Z = 4x + 3y$
(0, 0)	0
(0, 40)	120
(20, 40)	200
(60, 20)	300 ← Maximum
(60, 0)	240

Hence, maximum value of Z = 300 < 325

So, the quantity in column B is greater.

Q. 2. The feasible solution for a LPP is shown in given figure. Let $Z = 3x - 4y$ be the objective function Minimum of Z occurs at



(A) (0, 0) **(B)** (0, 8)
(C) (5, 0) **(D)** (4, 10)

Ans. Option (B) is correct.

Explanation:

Corner points	Corresponding value of $Z = 3x - 4y$
(0, 0)	0

(5, 0)	15 ← Maximum
(6, 5)	–2
(6, 8)	–14
(4, 10)	–28
(0, 8)	–32 ← Minimum

Hence, the minimum of Z occurs at (0, 8) and its minimum value is (–32).

Q. 3. Refer to Q.2 of multiple choice questions, maximum of Z occurs at

(A) (5, 0) **(B)** (6, 5)

(C) (6, 8) **(D)** (4, 10)

Ans. Option (A) is correct.

Explanation: Maximum of Z occurs at (5, 0).

Q. 4. Refer to Q.2 of multiple choice questions, (Maximum value of Z + Minimum value of Z) is equal to

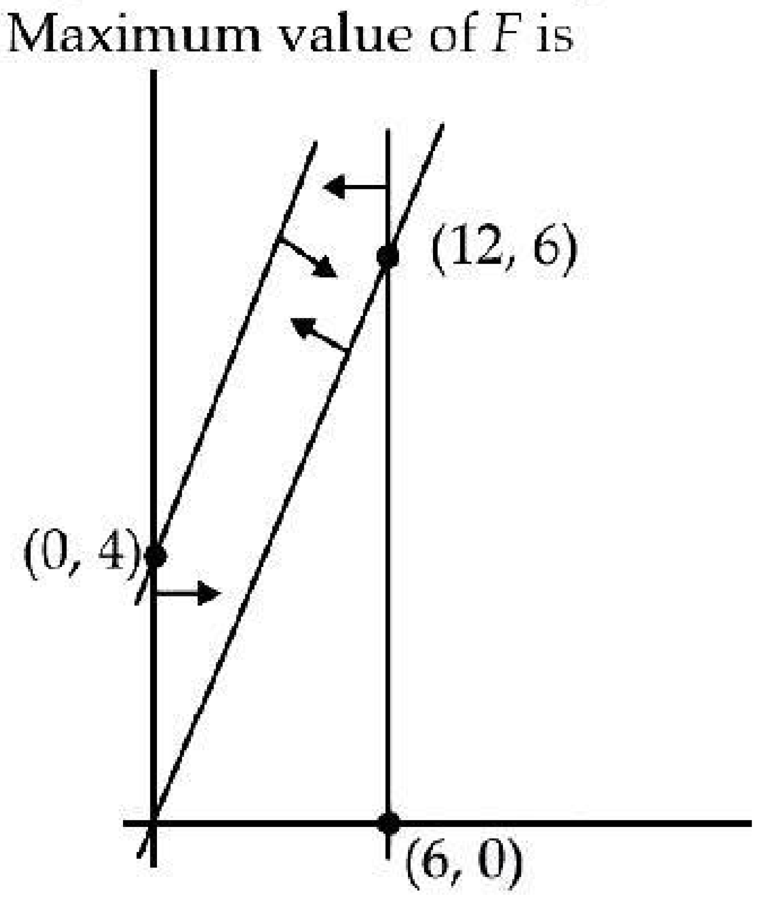
(A) 13 **(B)** 1

(C) 13 **(D)** 17

Ans. Option (D) is correct.

Explanation: Maximum value of Z + Minimum value of $Z = 15 - 32 = -17$

Q. 5. The feasible region for an LPP is shown in the given Figure. Let $F = 3x - 4y$ be the objective function. Maximum value of F is



(A) 0 **(B)** 8

(C) 12 **(D)** –18

Ans. Option (C) is correct.

Explanation: The feasible region as shown in the figure, has objective function $F = 3x - 4y$.

Corner points	Corresponding value of $F = 3x - 4y$
(0, 0)	0
(12, 6)	12 ← Maximum
(0, 4)	–16 ← Minimum

Hence, the maximum value of F is 12.

Q. 6. Refer to Q.5 of multiple choice questions, minimum value of F is

(A) 0 **(B)** –16

(C) 12 **(D)** does not exist

Ans. Option (B) is correct.

Explanation: Minimum value of F is -16 at (0, 4).

Q. 7. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let $F = 4x + 6y$ be the objective function.

The minimum value of F occurs at

(A) (0, 2) only

(B) (3, 0) only

(C) the mid-point of the line segment joining the points (0, 2) and (3, 0) only

(D) any point on the line segment joining the points (0, 2) and (3, 0)

Ans. Option (D) is correct.

Explanation :

Corner points	Corresponding value of $F = 4x+6y$
(0, 2)	12 ← Minimum
(3, 0)	12 ← Minimum
(6, 0)	24
(6, 8)	72 ← Maximum
(0, 5)	30

Hence, minimum value of F occurs at any points on the line segment joining the points (0, 2) and (3, 0).

Q. 8. Refer to Q. 7 above, Maximum of F – Minimum of F =

(A) 60 **(B)** 48

(C) 42 **(D)** 18

Ans. Option (A) is correct.

Explanation: Maximum of F – Minimum of

$$F = 72 - 12 = 60$$

Q. 9. Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the minimum of Z occurs at (3, 0) and (1, 1) is

(A) $p = 2q$ **(B)** $p = q/2$

(C) $p = 3q$ **(D)** $p = q$

Ans. Option (B) is correct.

Explanation :

Corner points	Corresponding value of $Z = px + qy$; $p, q > 0$
(0, 3)	$3q$
(1, 1)	$p + q$
(3, 0)	$3p$

So, condition of p and q, so that the minimum of Z occurs at (3, 0) and (1, 1) is

$$p+q = 3p$$

$$\Rightarrow \quad 2p = q$$

$$\therefore \quad p = \frac{q}{2}$$

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Q. 1. Assertion (A): Feasible region is the set of points which satisfy all of the given constraints and objective function too.

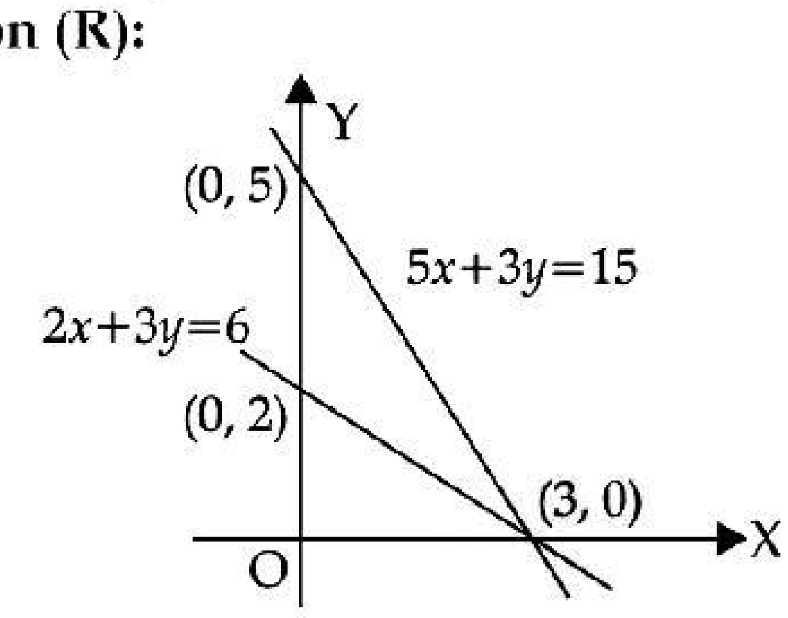
Reason (R): The optimal value of the objective function is attained at the points on X-axis only.

Ans. Option (C) is correct.

> *Explanation:* The optimal value of the objective function is attained at the corner points of feasible region.

Q. 2. Assertion (A): The intermediate solutions of constraints must be checked by substituting them back into objective function.

Reason (R):



Here (0, 2); (0, 0) and (3, 0) all are vertices of feasible region.

Ans. Option (D) is correct.

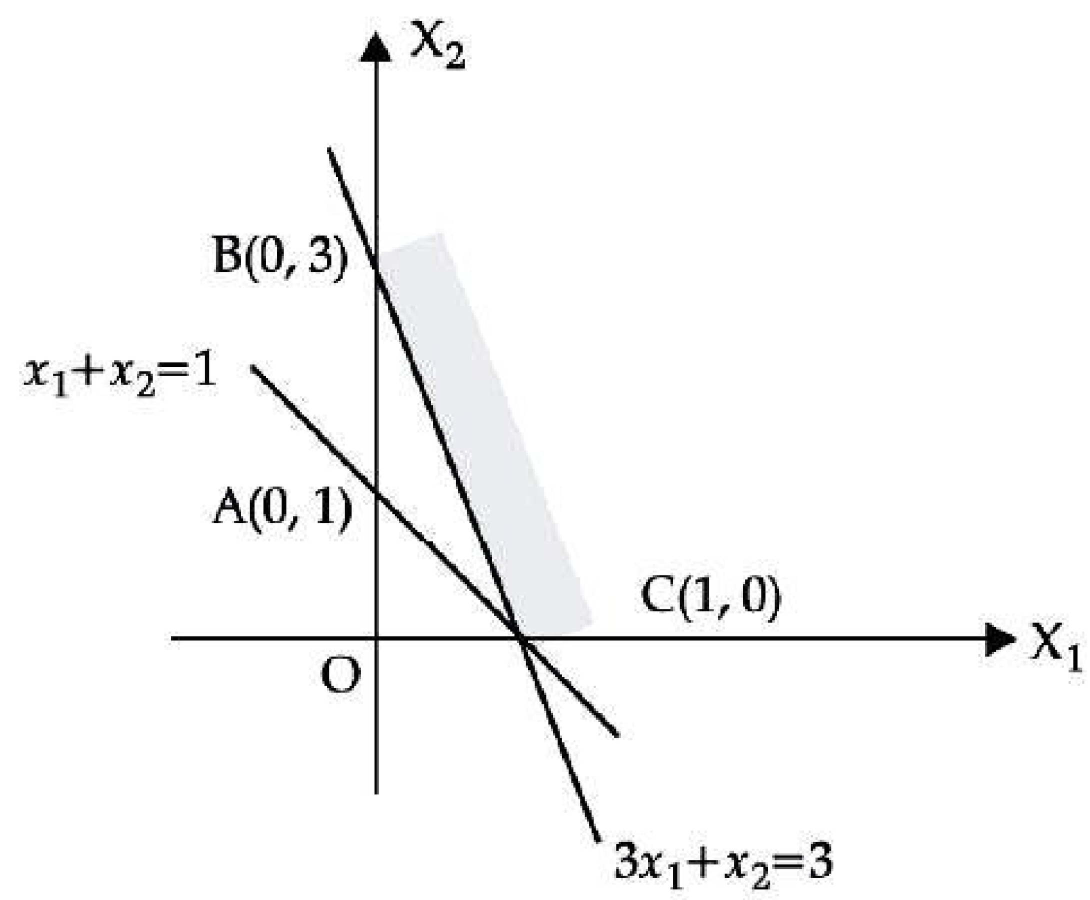
> *Explanation:* The intermediate solutions of constraints must be checked by substituting them back into constraint equations.

Q. 3. Assertion (A) : For the constraints of linear optimizing function $Z = x_1 + x_2$ given by $x_1 + x_2 \leq 1$, $3x_1 + x_2 \geq 1$, there is no feasible region.

Reason (R): $Z = 7x + y$, subject to $5x + y \leq 5$, $x + y \geq 3$, $x \geq 0$, $y \geq 0$. Out of the corner points of feasible region (3, 0), $\left(\frac{1}{2}, \frac{5}{2}\right)$, (7, 0) and (0,5), the maximum value of Z occurs at (7, 0).

Ans. Option (B) is correct.

> *Explanation:* Assertion (A) is correct.
> Clearly from the graph below that there is no feasible region.



Reason (R) is also correct.

Corner Points	$Z = 7x + y$
(3, 0)	21
$\left(\frac{1}{2}, \frac{5}{2}\right)$	6
(7, 0)	49 maximum
(0, 5)	5

Q. 4. Assertion (A): $Z = 20x_1 + 20x_2$, subject to $x_1 \geq 0$, $x_2 \geq 2$, $x_1 + 2x_2 \geq 8$, $3x_1 + 2x_2 \geq 15$, $5x_1 + 2x_2 \geq 20$. Out of the corner points of feasible region (8, 0), $\left(\frac{5}{2}, \frac{15}{2}\right)$, $\left(\frac{7}{2}, \frac{9}{4}\right)$ and (0,10), the minimum value of Z occurs at $\left(\frac{7}{2}, \frac{9}{4}\right)$.

Reason (R) :

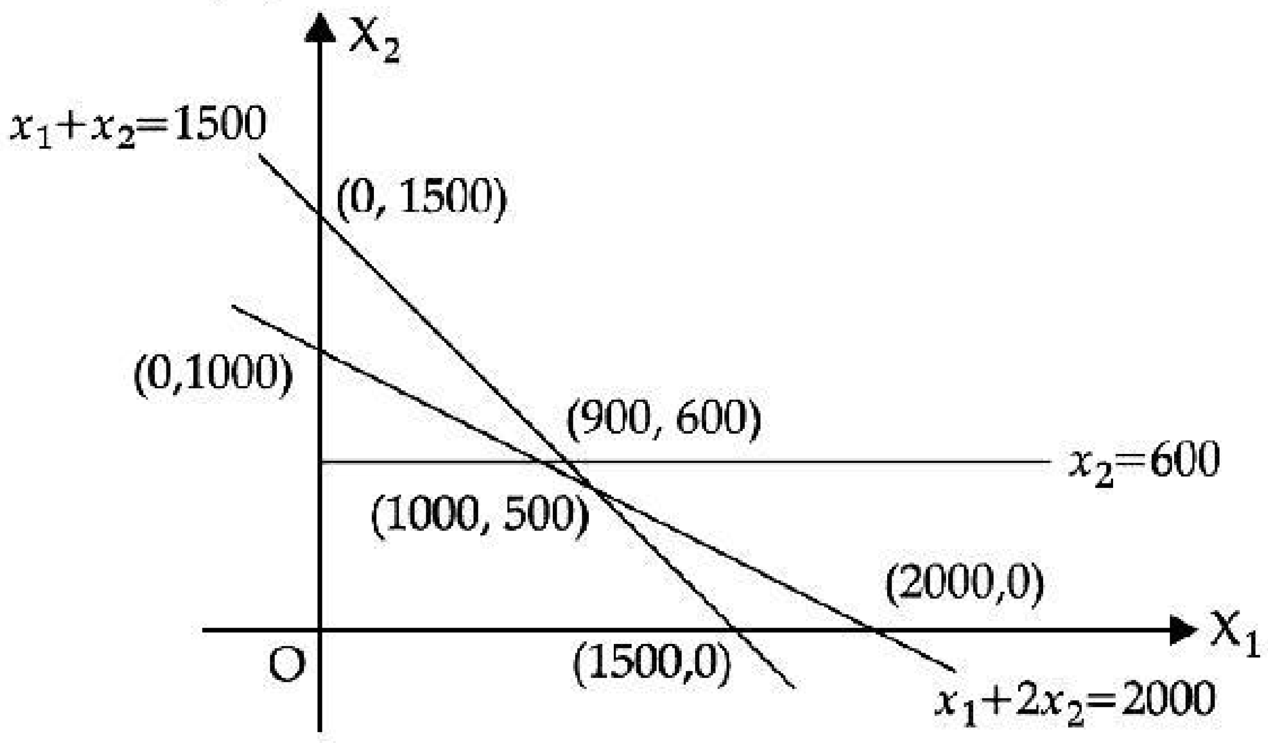
Corner Points	$Z = 20x_1 + 20x_2$
(8, 0)	160
$\left(\frac{5}{2}, \frac{15}{4}\right)$	125
$\left(\frac{7}{2}, \frac{9}{4}\right)$	115 minimum
(0, 10)	200

Ans. Option (A) is correct.

> *Explanation:* Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).

Q. 5. Assertion (A): For the constraints of a LPP problem given by
$x_1 + 2x_2 \leq 2000$, $x_1 + x_2 \leq 1500$, $x_2 \leq 600$ and $x_1, x_2 \geq 0$, the points (1000, 0), (0, 500), (2, 0) lie in the positive bounded region, but point (2000, 0) does not lie in the positive bounded region.

Reason (R):



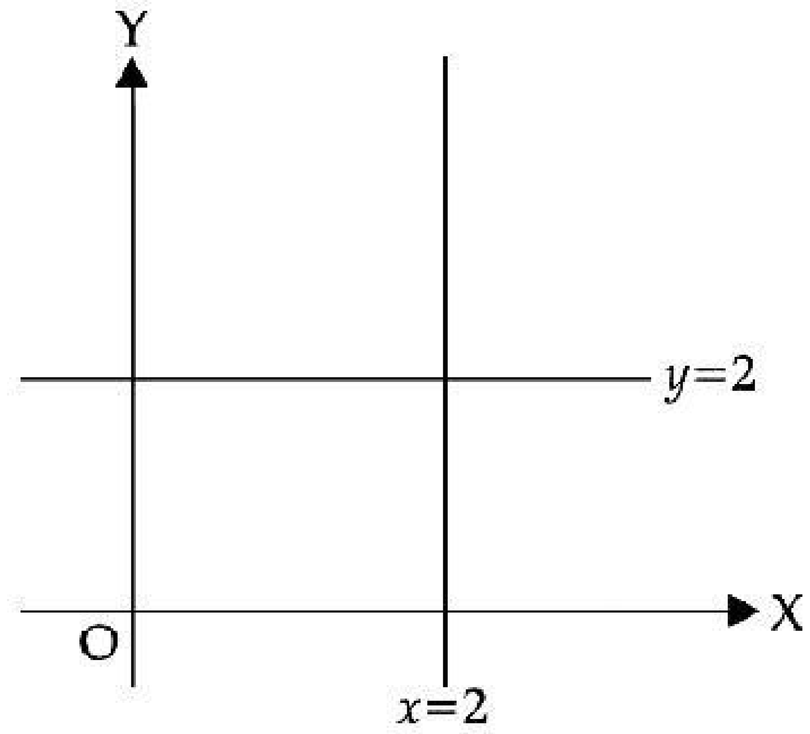
From the graph, it is clear that the point (2000, 0) is outside.

Ans. Option (A) is correct.

> *Explanation:* Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Q. 6. Assertion (A) : The graph of $x \leq 2$ and $y \geq 2$ will be situated in the first and second quadrants.

Reason (R):



Ans. Option (A) is correct.

> *Explanation:* It is clear from the graph given in the Reason (R) that Assertion (A) is true.

CASE-BASED MCQs

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:

An aeroplane can carry a maximum of 200 passengers. A profit of ₹ 1000 is made on each executive class ticket and a profit of ₹ 600 is made on each economy class ticket. The airline reserves at least 20 seats for the executive class. However, at least 4 times as many passengers prefer to travel by economy class, than by executive class. It is given that the number of executive class tickets is x and that of economy class tickets is y.

Q. 1. The maximum value of $x + y$ is_______.

(A) 100 (B) 200

(C) 20 (D) 80

Ans. Option (B) is correct.

Q. 2. The relation between x and y is _______.

(A) $x < y$ (B) $y > 80$

(C) $x \geq 4y$ (D) $y \geq 4x$

Ans. Option (D) is correct.

Q. 3. Which among these is not a constraint for this LPP?

(A) $x \geq 0$ (B) $x + y \leq 200$

(C) $x \geq 80$ (D) $4x - y \leq 0$

Ans. Option (C) is correct.

Q. 4. The profit when $x = 20$ and $y = 80$ is _______.

(A) ₹60,000 (B) ₹68,000

(C) ₹64,000 (D) ₹1,36,000

Ans. Option (B) is correct.

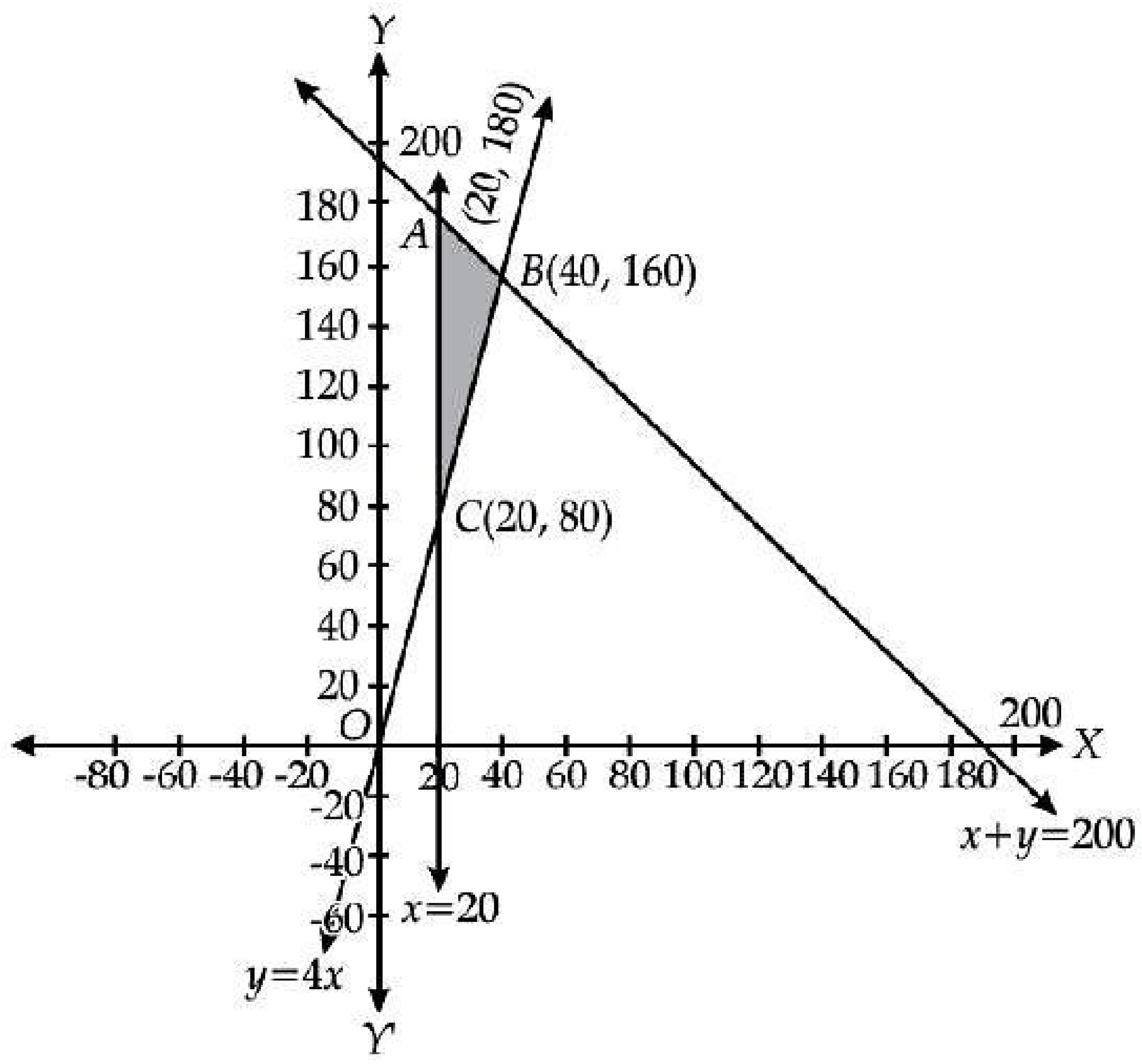
Q. 5. The maximum profit is ₹ _______.

(A) 1,36,000 (B) 1,28,000

(C) 68,000 (D) 1,20,000

Ans. Option (A) is correct.

> *Explanation:*
>
> Objective function :
>
> $$\text{Maximise } Z = 1000x + 600y$$
>
> Constraints:
>
> $$x + y \geq 200$$
> $$y \geq 20, x \geq 0$$
> $$y \geq 4x$$



The corner points are $A(20, 180)$, $B(40, 160)$, $C(20, 80)$

Evaluating the objective function

$Z = 1{,}000x + 600y$ at A, B and C

At $A(20, 180)$, $Z = 1{,}000 \times 20 + 600 \times 180$
$= 20{,}000 + 1{,}08{,}000$
$= ₹\ 1{,}28{,}000$

At $B(40, 160)$, $Z = 1{,}000 \times 40 + 600 \times 160$
$= 40{,}000 + 96{,}000$
$= ₹\ 1{,}36{,}000$ (max.)

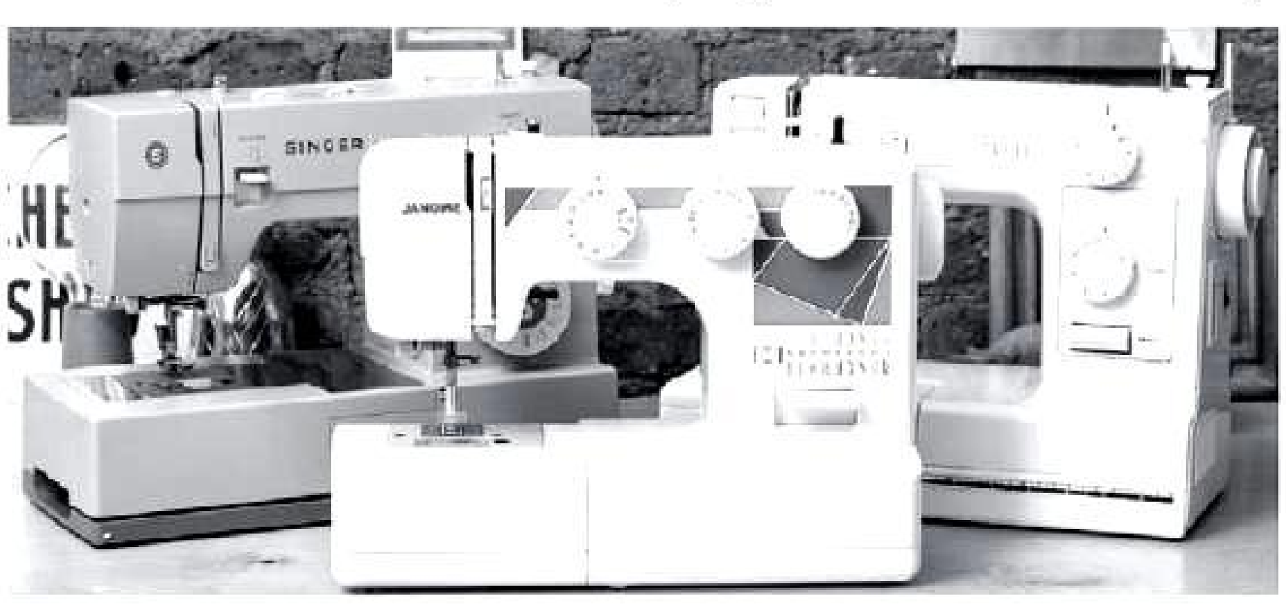
At $C(20, 80)$, $Z = 1000 \times 20 + 600 \times 80$
$= 20{,}000 + 48{,}000$
$= ₹\ 68{,}000$

or Z is maximum, when $x = 40$, $y = 160$.

or 40 tickets of executive class and 160 tickets of economy class should be sold to get the maximum profit of ₹ 1,36,000.

II. Read the following text and answer the following questions on the basis of the same:

A dealer in rural area wishes to purchase a number of sewing machines. He has only ₹5,760 to invest and has space for at most 20 items for storage. An electronic sewing machine cost him ₹360 and a manually operated sewing machine ₹240. He can sell an electronic sewing machine at a profit of ₹22 and a manually operated machine at a profit of ₹18. Assume that the electronic sewing machines he can sell is x and that of manually operated machines is y.



Q. 1. The objective function is _______.

(A) Maximise $Z = 360x + 240y$

(B) Maximise $Z = 22x + 18y$

(C) Minimise $Z = 360x + 240y$

(D) Minimise $Z = 22x + 18y$

Ans. Option (B) is correct.

Q. 2. The maximum value of $x + y$ is _______.

(A) 5760 **(B)** 18

(C) 22 **(D)** 20

Ans. Option (D) is correct.

Q. 3. Which of the following is not a constraint?

(A) $x + y \geq 20$

(B) $360x + 240y \leq 5{,}760$

(C) $x \geq 0$

(D) $y \geq 0$

Ans. Option (A) is correct.

Q. 4. The profit is maximum when $(x, y) =$ _______.

(A) (5, 15) **(B)** (8, 12)

(C) (12, 8) **(D)** (15, 5)

Ans. Option (B) is correct.

Q. 5. The maximum profit is ₹ _______.

(A) 5,760 **(B)** 392

(C) 362 **(D)** 290

Ans. Option (B) is correct.

Explanation:

Objective function :

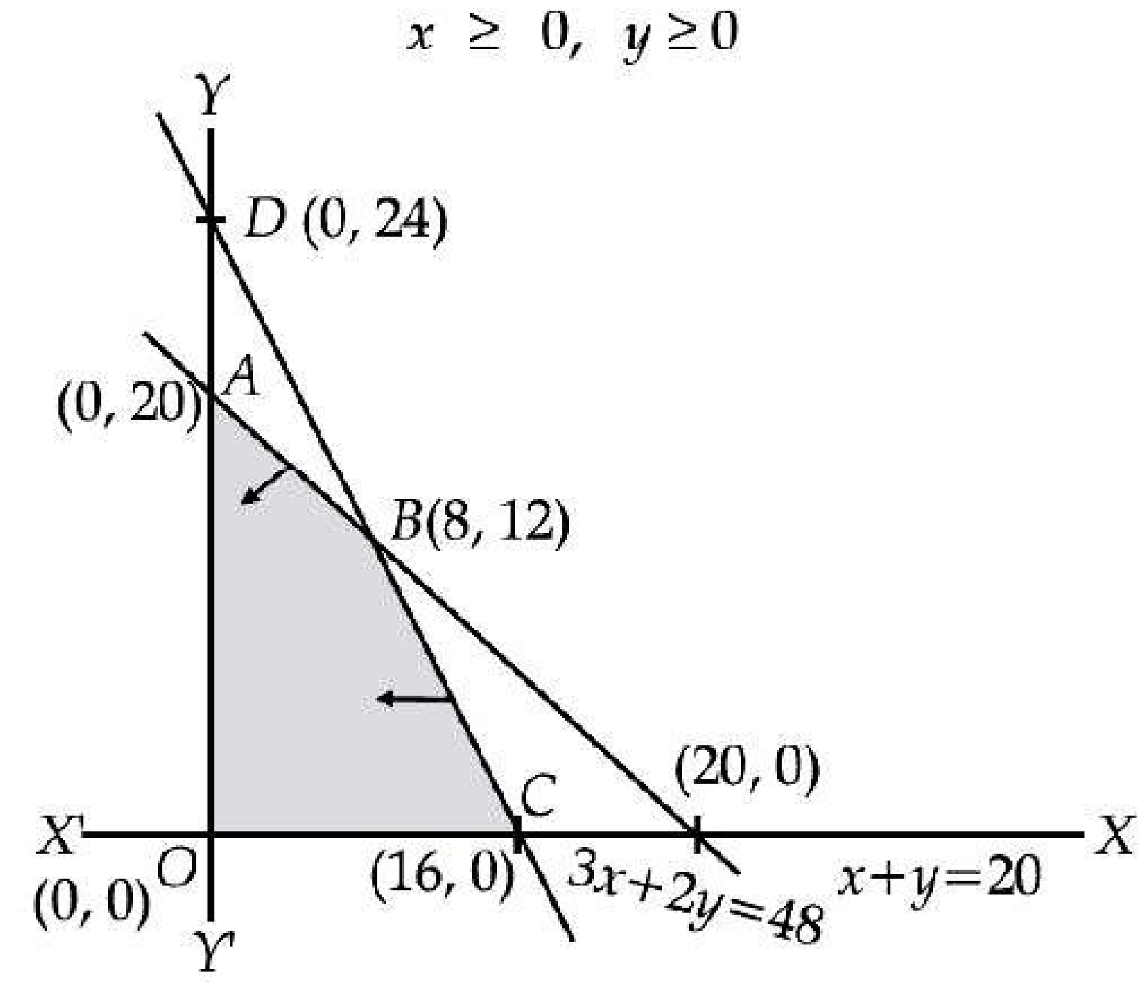
Maximise $Z = 22x + 18y$

Constraints :

$$x + y \leq 20$$
$$360x + 240y \leq 5{,}760$$

Or $3x + 2y \leq 48$

$$x \geq 0, \ y \geq 0$$



Vertices of feasible region are :

$A(0, 20)$, $B(8, 12)$, $C(16, 0)$ & $O(0, 0)$

$P(A) = 360$, $P(B) = 392$, $P(C) = 352$

∴ For Maximum P, Electronic machines = 8, and Manual machines = 12. Max. profit ₹392

III. Read the following text and answer the following questions on the basis of the same:

A fruit grower can use two types of fertilizer in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash, and chlorine in a bag of each brand are given in the table. Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine.

	kg per bag	
	Brand P	**Brand Q**
Nitrogen	3	3.5
Phosphoric acid	1	2
Potash	3	1.5
Chlorine	1.5	2

Q. 1. The Objective function to minimise the amount of nitogen added to garden?

(A) Maximise $Z = 3x + 4y$
(B) Minimise $Z = 3x + 3.5y$
(C) Maximise $Z = 4x + 3.5y$
(D) Minimise $Z = 3x + 4y$

Ans. Option (B) is correct.

Q. 2. If the grower wants to minimise the amount of nitrogen added to the garden, how many bags of brand P should be used?

(A) 40 (B) 50
(C) 100 (D) 60

Ans. Option(A) is correct.

Q. 3. If the grower wants to minimise the amount of nitrogen added to the garden, how many bags of brand Q should be used?

(A) 40 (B) 50
(C) 100 (D) 60

Ans. Option (C) is correct.

Q. 4. What is the minimum amount of nitrogen added in the garden?

(A) 595 kg (B) 550 kg
(C) 400 kg (D) 470 kg

Ans. Option (D) is correct.

Q. 5. What is the total number of bags used by fruit grower to minimise the amount of nitrogen ?

(A) 160 (B) 190
(C) 140 (D) 130

Ans. Option (C) is correct.

Explanation: Let the fruit grower use x bags of brand P and y bags of brand Q.

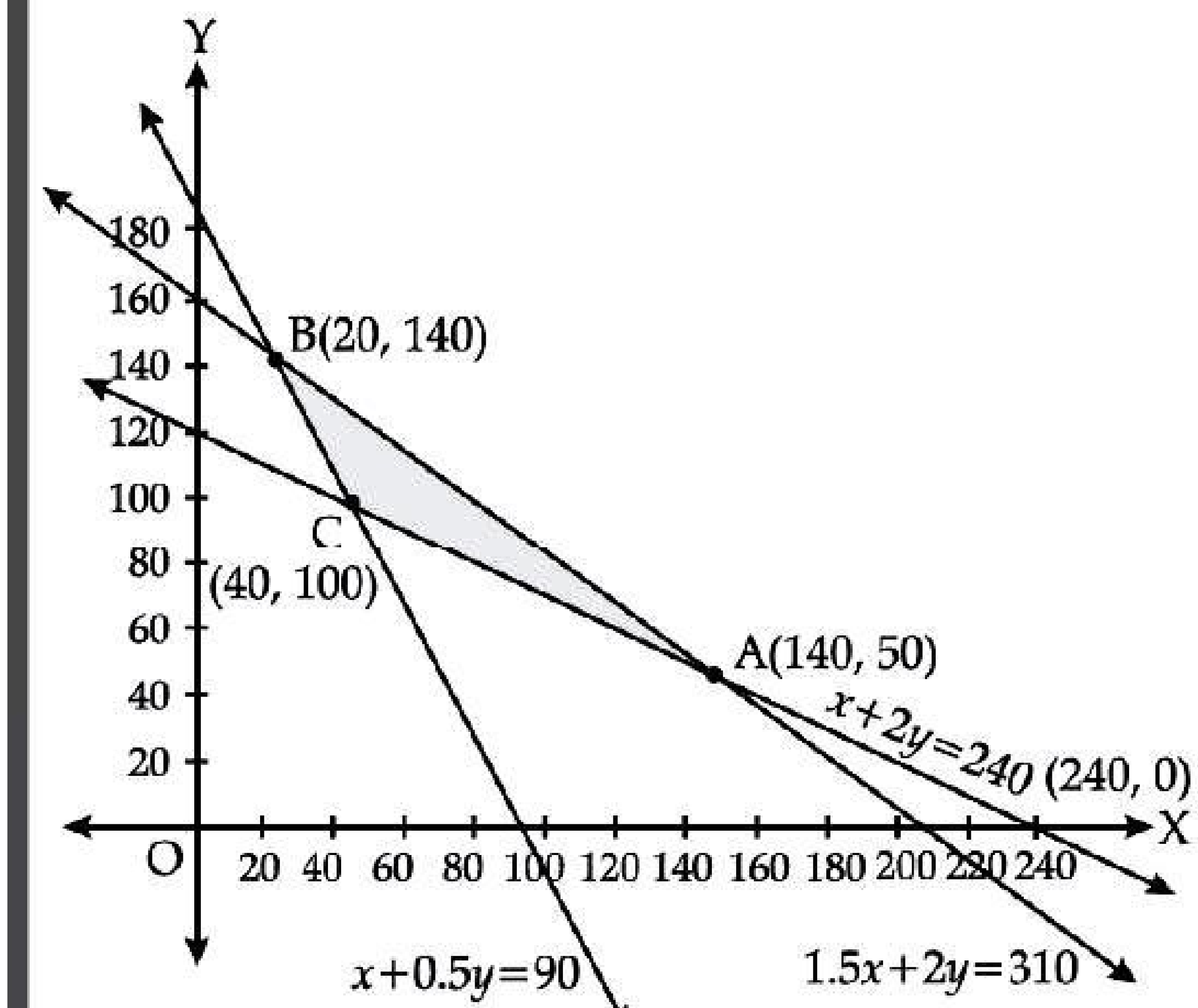
The problem can be formulated as follows:

Minimise $Z = 3x + 3.5y$ ...(i)

Subject to the constraings,

$$x + 2y \geq 240 \quad ...(ii)$$
$$x + 0.5y \geq 90 \quad ...(iii)$$
$$1.5x + 2y \leq 310 \quad ...(iv)$$
$$x, y \geq 0 \quad ...(v)$$

The feasible region determined by the system of constraints is as follows:



The corner points are $A(240, 50)$, $B(20, 140)$, and $C(40, 100)$

Corner points	$Z = 3x + 3.5y$	
$A(140, 50)$	595	
$B(20, 140)$	550	
$C(40, 100)$	470	← Minimum

The minimum value of Z is 470 at (40, 100).

Thus, 40 bags of brand P and 100 bags of brand Q should be added to the garden to minimise the amount of nitrogen.

The minimum amount of nitrogen added to the garden is 470 kg.